1

Basic methods

1.1 When we add and when we subtract

We start our course in enumerative combinatorics by considering a very simple task, namely finding the size of the union of two disjoint sets.

1.1.1 When we add

A group of friends went on a canoe trip. Five of them fell into the water at one point or another during the trip, while seven completed the trip without even getting wet. How many friends went on this canoe trip?

Before the reader laughs at us for starting the book with such a simple question, let us give the answer. Of course, $5 + 7 = 12$ people went on this trip. However, it is important to point out that such a simple answer was only possible because each person on the trip *either* fell into the water *or* stayed dry. There was no middle way, there was no way to belong to both groups, or to neither group. Once you fall into the water, you know it. In other words, each person was included in exactly one of those two groups of people.

In contrast, let us assume that we are not told how many people did or did not fall into the water, but instead are told that five people wore white shirts on this trip and eight people wore brown hats. Then we could not tell how many people went on this trip, as there could be people who belonged to *both groups* (it is possible to wear both a white shirt and a brown hat), and there could be people who belonged to neither.

We can now present the first, and easiest, counting principle of this book. Let $|X|$ denote the number of elements of the finite set X. So, for instance, $|\{2, 3, 5, 7\}| = 4$. Recall that two subsets are called *disjoint* if they have no elements in common.

Theorem 1.1 (Addition principle) *If A and B are two* disjoint *finite sets, then*

$$|A \cup B| = |A| + |B|. \tag{1.1}$$

It may seem somewhat strange that we provide proof for such an extremely simple statement, but we want to set standards.

Proof: Both sides of (1.1) count the elements of the same set, the set $A \cup B$.

DOI: 10.1201/9781003304272-1