he either chooses a sundae, in one of five ways, or a coffee and a cookie, in $4 \cdot 2 = 8$ ways. So he has $5 + 8 = 13$ choices for dessert. Therefore, if dessert is considered the third course, he has $4 \cdot 5 \cdot 13 = 260$ choices, in agreement with what we computed above. $\Diamond$

Example 1.19 *A college senior will spend her weekend visiting some graduate schools. Because of geographical constraints, she can either go to the north, where she can visit four schools out of the ten schools in which she is interested, or she can go to the south, where she can visit five schools out of eight schools in which she is interested. How many different itineraries can she set up?*

Note that we are interested in the number of possible itineraries, so the order in which the student visits the schools is important.

Solution: (of Example 1.19) The student can either go to the north, in which case, by Theorem 1.17, she will have $(10)_4$ possibilities, or she can go to the south, in which case she will have $(8)_5$ possibilities. Therefore, by the addition principle, the total number of possibilities is

$$(10)_4 + (8)_5 = 10 \cdot 9 \cdot 8 \cdot 7 + 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 5040 + 6720 = 11760.$$

$\Diamond$

Quick Check

1. How many three-digit positive integers are there in which all digits are divisible by 3?
2. How many three-digit positive integers are there in which all digits are divisible by 3 and no digit is repeated?
3. What is the number of permutations of the digits 1, 2, 3, 4, 5, and 6 in which each digit in an even position is larger than each digit in an odd position?

1.3 When we divide

We sometimes make choices that look different, but for the purposes of our current counting problem, they are not really different. We have to take this into account so that we do not count the same choice more than once.