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Counting magic squares and magic cubes

In earlier chapters, we considered various distribution problems. Now we are returning to that topic, but our distribution problems will now be, in a sense, multidimensional.

10.1 A distribution problem

Let us assume that we have to distribute 60 building blocks among three children. Let us say that 20 of the blocks are red, 20 are blue, and 20 are green. To preserve world peace, it is our intention to give 20 blocks to each child. In how many different ways can this be done if blocks of the same color are identical? (Note that the problem is not too interesting if even blocks of the same color are different; indeed, in that case all blocks are different, and the answer is $\binom{60}{20} \cdot \binom{40}{20}$.)

This distribution problem is more difficult than the ones we have seen in earlier chapters. In order to understand the problem better, we will represent a possible distribution by a square grid, as shown in [Figure 10.1](#).

	Red	Blue	Green
Miki	7	5	8
Benny	6	8	6
Vinnie	7	7	6

Figure 10.1

A possible distribution.

Note that the sum of the elements in each row is equal to 20 (as each child gets 20 blocks), and the sum of the elements in each column is equal to 20, since there are 20 blocks of each color. This observation motivates the following definition.