Quick Check

1. Let P_1 and P_2 be two paths of maximum length in a tree T. Prove that P_1 and P_2 have at least one vertex in common.
2. What is the total number of leaves in all trees on vertex set $[n]$?
3. Find the number of all trees on vertex set $[n]$ in which exactly two vertices have degree higher than 1. sequence h_n.

5.2 Graphs and functions

We have seen that the number of all trees on vertex set $[n]$ is equal to the number of all functions $f : [n-2] \to [n]$. This is not the only identity connecting the enumeration of functions to the enumeration of graphs.

All functions in this section will be functions $f : [n] \to [n]$. We will refer to this fact by saying that they are functions *on* $[n]$.

5.2.1 Acyclic functions

A function $f : [n] \to [n]$ is called *acyclic* if its short diagram does not contain cycles except for loops. Loops must be allowed, since the short diagram of f contains n edges, and so the short diagram cannot be cycle free.

Example 5.20 *Let $n = 6$. Then the function $f : [n] \to [n]$ defined by $f(i) = i+1$ if i is not divisible by 3, and $f(i) = i$ otherwise, is acyclic, as we can see in Figure 5.21.*

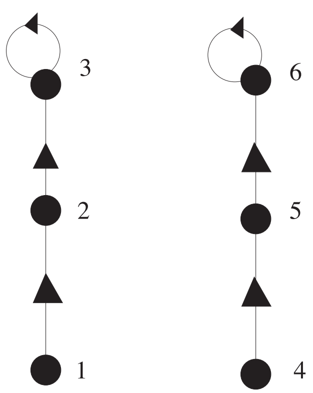

Figure 5.21
An acyclic function on [6].