Appendix

The method of mathematical induction

This appendix is meant to provide first aid for students who have not seen proofs with the method of mathematical induction before. While this method is based on a simple idea, it takes practice to be able to successfully apply it. Therefore, after reading our quick review of the method, the reader should consult any textbook on the transition to higher mathematics, such as [78] for a more detailed treatment of the method or [12] for a chapter on combinatorial applications of the method.

A.1 Weak induction

If a statement is true today, and from the fact that it is true on one day follows the fact that it will be true the following day, then we can conclude that the statement will be true every day from now on. An example of this is the statement "the 2004 football season is over."

This idea is the centerpiece of a very powerful, and very often used, tool in proving mathematical statements, that of *mathematical induction*. A straightforward translation of the above idea to a more mathematical context is as follows: Let us assume that a statement involving the variable n is true for $n = 1$, and from the fact that it is true for $n = k$ follows the fact that it is also true for $n = k + 1$. Then the statement is true for all positive integers n.

In practice, it is usually easy to prove that the statement is true for $n = 1$ (the initial step), but it is somewhat harder to prove that the fact that the statement is true for $n = k$ implies the fact that it is also true for $n = k + 1$. Once we succeed in proving these two "local" statements, the method of induction will imply that the original "global" statement is true for all positive integers n.

Example A.1 *For all positive integers n,*

$$\sum_{i=1}^{n} i(i-1) = \frac{(n+1)n(n-1)}{3}. \tag{A.1}$$

Proof: We prove our statement by induction on n.

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